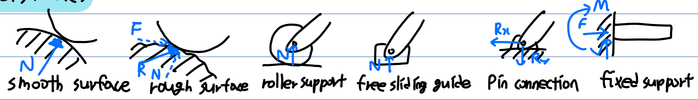
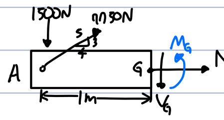
Ch0. Statics



ex) Equilibrium of deformable body



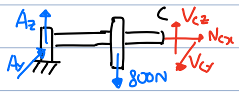
$\Sigma F_x = 7750 \times \frac{4}{5} + N_G = 0 \rightarrow N_G = -6200N$

$\Sigma F_y = -1500N + 7750N \times \frac{3}{5} - V_G = 0 \rightarrow V_G = 3150N$

$\circlearrowleft \Sigma M_G = M_G - 7750N \times \frac{3}{5} \times 1m + 1500N \times 1m = 0 \rightarrow M_G = 3150 N\cdot m$

Ch1. Stress

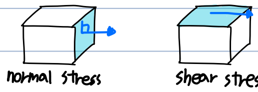
ex1) internal loading on cross-section



$\therefore V_{cz} = 800N - A_z$

$\therefore V_{cy} = -A_y$

$\therefore N_c = 0$



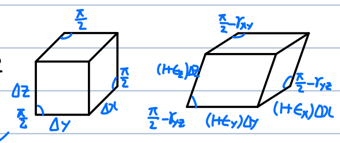
Saftey factor: $F.S = \frac{\sigma_{fail}}{\sigma_{allow}}$ or $\frac{\tau_{fail}}{\tau_{allow}}$

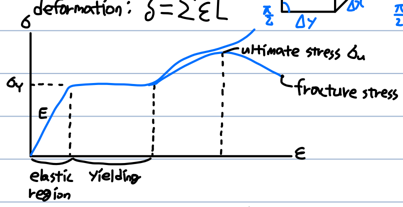
Ch2. Strain

Strain은 무차원수

normal strain: $\varepsilon = \frac{L - L_0}{L_0}$

deformation: $\delta = \Sigma \varepsilon L$





elastic region: $\sigma = E\varepsilon$ (E: Young's Modulus), 원래 형상으로 돌아올 수 있는 영역

Yielding: yield strength (σ_Y)

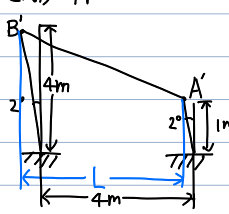
resillence: $u = \frac{1}{2}\sigma\varepsilon$

toughness: $u_t = \int \sigma \varepsilon$

Poisson's Ratio: $\nu = -\frac{\varepsilon_{lat}}{\varepsilon_{long}}$

Shear Modulus: $G = \frac{E}{2(1+\nu)}$

ex1) approximate with the normal strain



$L = 4 \times \frac{2\pi}{180} + 4 - 1 \times \frac{2\pi}{180} = 4.1047m$

$\overline{A'B'} = \sqrt{3^2 + 4.1047^2} = 5.0841m$

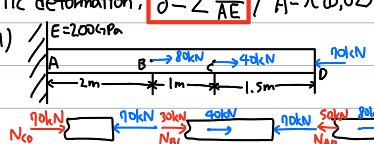
$\overline{AB} = 5m$

$\therefore \varepsilon = \frac{\overline{A'B'} - \overline{AB}}{\overline{AB}} = 0.0168 \, (m/m)$

Ch04. Axial Load

Elastic deformation: $\boxed{\delta = \Sigma \frac{NL}{AE}}$ / $A = \pi(0.025m)^2$, $E = 200 \times 10^9 N/m^2$

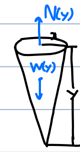
ex1)



$\delta_D = \delta_{A/D} = \delta_{C/D} + \delta_{B/C} + \delta_{A/B} = \frac{1}{AE}(N_{CD}L_{CD} + N_{BC}L_{BC} + N_{AB}L_{AB})$

$= \frac{1}{AE}(-70kN \times 1.5m - 30kN \times 1m + 50kN \times 2m)$

ex2) $\gamma = 6kN/m^3$, $r_0 = 0.3m$, $L = 3m$, $E = 9GPa$



$\frac{x}{y} = \frac{r_0}{L} \rightarrow x = \frac{r_0}{L}y = 0.1y$

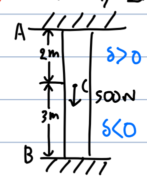
$V = \frac{\pi}{3}yx^2 = \frac{\pi r_0^2}{3L^2}y^3 = 0.01047y^3$

$W = \gamma V = 62.83y^3$, $\Sigma F_y = N(y) - W(y) = 0 \rightarrow N(y) = 62.83y^3$

$A(y) = \pi x^2 = \frac{\pi r_0^2}{L^2}y^2 = 0.03142y^2$

$\delta = \int_0^L \frac{N(y)dy}{A(y)E} = \frac{\gamma}{3E}\int_0^L y\,dy = \frac{\gamma L^2}{6E} = 10^{-6}m = 1\mu m$

Statically Indeterminate problem (부정정 문제)



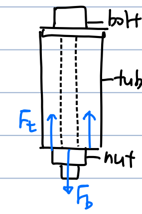
$F_B + F_A - P = 0$, $\delta_{A/B} = 0$

$\frac{F_A L_{AC}}{AE} - \frac{F_B L_{CB}}{AE} = 0$

$F_A = F_B(L_{CB}/L_{AC})$, $F_A + F_B = P$

$\rightarrow F_A = 300N$, $F_B = 200N$

ex3)



F_t: 압축, F_b: 인장

$\Sigma F_y = F_t - F_b = 0 \rightarrow F_t = F_b$

$\delta = \delta_t + \delta_b = 0.5mm$

$\frac{F_t \times 0.06m}{2.3562 \times 10^{-4}m^2 \times 45.7 \times 10^9 Pa} + \frac{F_t \times 0.06m}{7.854 \times 10^{-5}m^2 \times 73.1 \times 10^9 Pa} = 5 \times 10^{-4}m$

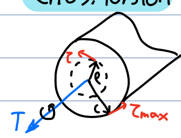
$0.5697 \times 10^{-8}F_t + 1.0451 \times 10^{-8}F_t = 5 \times 10^{-4}$

$\therefore F_t = F_b = 30.96kN$

· Thermal stress

$\delta_T = \alpha \Delta T L$

Ch05. Torsion



$\boxed{\tau = \frac{T\rho}{J}}$, $\tau_{max} = \frac{Tc}{J}$

$J = \frac{\pi}{2}c^4$ or $J = \frac{\pi}{2}(c_o^4 - c_i^4)$

· Power: $P = T\omega = 2\pi f T$

ex1) 5hp, 175rpm, $\tau_{allow} = 100MPa$

$P = 5hp\left(\frac{746W}{1hp}\right) = 3730W$

$\omega = \frac{175rev}{min}\left(\frac{2\pi rad}{1rev}\right)\left(\frac{1min}{60s}\right) = 18.33 rad/s$

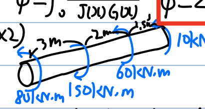
$T = \frac{P}{\omega} = 203.54 N\cdot m$

$\tau_{allow} = \frac{Tc}{J}$, $c = \frac{\tau_{allow} \times \frac{\pi}{2}c^4}{T}$, $c = \left(\frac{2T}{\pi \tau_{allow}}\right)^{1/3}$

· Angle of twist

$\phi = \int_0^L \frac{T(x)dx}{J(x)G(x)}$ $\boxed{\phi = \Sigma \frac{TL}{JG}}$

ex2)



$\phi_{A/D} = \frac{(+80kN\cdot m)L_{AB}}{JG} + \frac{(-70kN\cdot m)L_{BC}}{JG} + \frac{(-10kN\cdot m)L_{CD}}{JG}$

· Gear problem

거리비에 비례해 토크도 감소/증가 $T_A r_A = T_B r_B$

거리비에 비례해 Angle 감소/증가 $\phi_A r_A = \phi_B r_B$

ex3) $T_A = 45N\cdot m$, $L_{AB} = 2m$, $L_{CO} = 1.5m$, $r_B = 0.15m$, $r_c = 0.075m$, $d = 0.02m$

$T_D = \frac{r_c}{r_B}T_A = 22.5N\cdot m$, $J = \frac{\pi}{2}(0.01m)^4 = 1.571 \times 10^{-8}m^4$, $G = 80 \times 10^9 N/m^2$

$\phi_c = \frac{T_D L_{DC}}{JG} = \frac{22.5N\cdot m \times 1.5m}{1.571 \times 10^{-8}m^4 \times 80 \times 10^9 N/m^2} = 0.0269 rad$

$\phi_B r_B = \phi_c r_c$, $\phi_B = \frac{0.075m}{0.15m} \times 0.0269 rad = 0.0134 rad$

$\phi_{A/B} = \frac{T_A L_{AB}}{JG} = \frac{45N\cdot m \times 2m}{1.571 \times 10^{-8}m^4 \times 80 \times 10^9 N/m^2} = 0.0716 rad$

$\phi_{A/B} = \phi_A - \phi_B$, $\phi_A = \phi_{A/B} + \phi_B = 0.0850 rad$

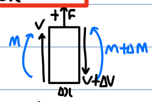
· Stress Concentration: $\tau_{max} = K\frac{Tc}{J}$

Ch06, Bending

- Shear and moment diagrams

$\boxed{\frac{dM}{dx}=V(x),\ \frac{dV}{dx}=w(x)}$ $\quad W \underset{\frac{d}{dx}}{\overset{\int dx}{\rightleftarrows}} V \underset{\frac{d}{dx}}{\overset{\int dx}{\rightleftarrows}} M$

direction



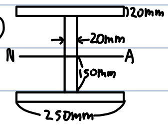
- Bending moment

$\sigma=-\frac{My}{I},\ \sigma_{max}=\frac{Mc}{I}$

- moment of inertia

$I=\Sigma(\bar{I}+Ad^2)$

ex1)



$I=\Sigma(\bar{I}+Ad^2)$

$=2\left[\frac{1}{12}0,25\times0,02^3+0,25\times0,02\times0,16^2\right]+\frac{1}{12}\times0,02\times0,3^3$

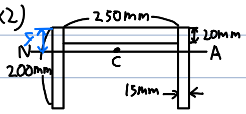
$=301,3\times10^{-6}m^4$

$\sigma_{max}=\frac{Mc}{I}=\frac{22,5\times10^3N\times0,17m}{301,3\times10^{-6}m^4}=12,7MPa$

- Centroid

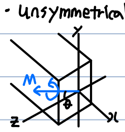
$\bar{y}=\frac{\Sigma\tilde{y}A}{\Sigma A}$

ex2)



$\bar{y}=\frac{2\times[0.1\times(0.2\times0.015)]+0,01\times(0,02\times0,25)}{2\times(0.2\times0,015)+(0,02\times0,25)}=59,09mm$

- unsymmetrical bending



$M_z=M\cos\theta,\ M_y=M\sin\theta$

$\boxed{\sigma=-\frac{M_zy}{I_z}+\frac{M_yz}{I_y}}$

ex3) $M=520N\cdot m,\ \cos\theta=\frac{12}{13},\ \sin\theta=\frac{5}{13}$

$M_z=M\cos\theta=480N\cdot m,\ M_y=M\sin\theta=200N\cdot m$

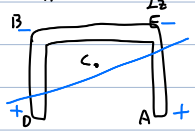
$\bar{y}=\frac{\Sigma\tilde{y}A}{\Sigma A}=\frac{[0.1m\times(0,02m\times0.2m)]\times2+0,01m\times0,36m\times0,02m}{0.02m\times0.2m\times2+0.36m\times0,02m}=0.05737m=57,37mm$

$I_y=\frac{1}{12}\times0.36m\times(0,02m)^3+(0,36m\times0,02m)\times(0.04737m)^2+2\times\left[\frac{1}{12}\times0,02m\times(0,2m)^3+(0,02m\times0,2m)\times(0.04263m)^2\right]$

$=5,760\times10^{-5}m^4$

$I_z=2\times\left[\frac{1}{12}\times0,2m\times(0,02m)^3+(0,2m\times0,02m)\times(0,19m)^2\right]+\frac{1}{12}\times0.02m\times(0,36m)^3=3,668\times10^{-4}m^4$

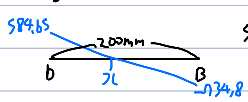
$\sigma_A=\sigma_{max}=\frac{M_zy}{I_z}+\frac{M_yz}{I_y}=\frac{480N\cdot m\times0.14263m}{3,668\times10^{-4}m^4}+\frac{200N\cdot m\times0,19m}{5,760\times10^{-5}m^4}=188647.76Pa+659722,22Pa=\underline{848,37kPa}$



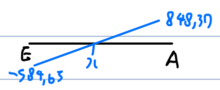
$\sigma_E=\frac{480N\cdot m\times0,05737m}{3,668\times10^{-4}m^4}-\frac{200N\cdot m\times0,19m}{5,760\times10^{-5}m^4}=75075,25Pa-659722,22Pa=-584,65kPa$

$\sigma_B=-\frac{480N\cdot m\times0,05737m}{3,668\times10^{-4}m^4}-\frac{200N\cdot m\times0,19m}{5,760\times10^{-5}m^4}=-75075,25Pa-659722,22Pa=-734,80kPa$

$\sigma_D=584,65kPa$



$584,65-6597,25x=0\rightarrow x=0,0886m=88,6mm$



$-584,65+7165,1x=0\rightarrow x=0,0816m=81,60mm$

$\frac{y}{z}=\frac{200-88,6-81,6}{400}=0,0745=\tan\theta$

$\therefore \underline{\theta=0,0744rad}$

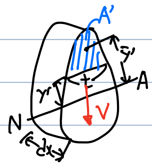
Ch07. Transverse Shear

· Shear formula

$\bar{y}=\frac{\Sigma A d}{\Sigma A}$

$Q=\int_{A'} y\,dA'=\bar{y}'A'$

$\tau=\frac{VQ}{It}$



· Shear flow

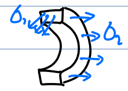
: $q=\frac{VQ}{I}$, 빔의 단위길이에 대한 힘

$F=q[N/m]\times s[m]$

Ch08. Combined Loadings

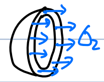
· Thin-walled pressure vessels

$\sigma_1=\frac{Pr}{t}$, $\sigma_2=\frac{Pr}{2t}$



· Spherical Vessels

$\sigma=\frac{Pr}{2t}$



· Stress caused by combined loadings

normal force: $\sigma=\frac{P}{A}$

Shear force: $\tau=\frac{VQ}{It}$

Bending moment: $\sigma=-\frac{My}{I}$

Torsional moment: $\tau=\frac{T\rho}{J}$

#8.20 $t=0.02m$, $P=3kN$, $d=0.15m$, $h=0.2m$

$A=ht=0.004m^2$, $I=\frac{1}{12}th^3=1.33\times10^{-5}m^4$

$M=(d-\frac{h}{2})\times P=150N\cdot m$

$\sigma_N=\frac{P}{A}=750kPa$, $\sigma_m=\frac{My}{I}=\frac{150N\cdot m\times0.05m}{1.33\times10^{-5}m^4}=1691729\ Pa$

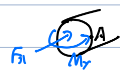
$\sigma_{max}=\sigma_N+\sigma_m=2.44MPa$, $\sigma_{min}=\sigma_N-\sigma_m=-941.72kPa$

#8.42 $d=0.04m$, $G_x=1500N$, $G_y=-600N$, $G_z=800N$, $M_x=-100N\cdot m$

$F_x=1500N$, $F_y=600N$, $F_z=-800N$

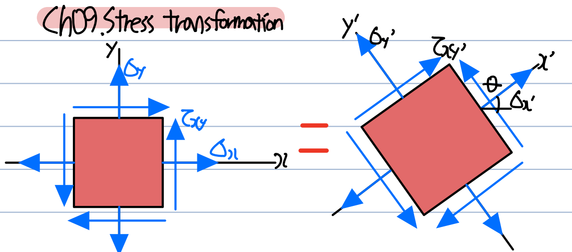
$M_y=0.3m\times800N=240N\cdot m$, $M_z=0.3m\times600N=180N\cdot m$

$I=\frac{\pi d^4}{64}=1.257\times10^{-7}m^4$, $A=\frac{\pi}{4}d^2=1.257\times10^{-3}m^2$



$\sigma_x=\frac{F_x}{A}+\frac{M_y\cdot\frac{d}{2}}{I}=\frac{1500N}{1.257\times10^{-3}m^2}+\frac{240N\cdot m\times0.02m}{1.257\times10^{-7}m^4}=39.38MPa$

Ch09. Stress transformation



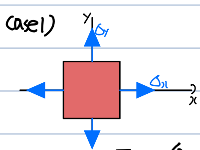
$\sigma_{avg}=\frac{\sigma_x+\sigma_y}{2}$, $R=\sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\tau_{xy}^2}$, $\tan2\theta_p=\frac{\tau_{xy}}{(\sigma_x-\sigma_y)/2}$

· Principal stress; $\sigma_{1,2}=\sigma_{avg}\pm R$

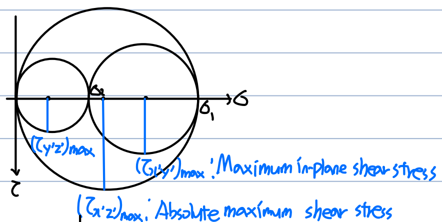
-Maximum In-plane stress: $\tau_{\substack{max\\ \text{in-plane}}}=\sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\tau_{xy}^2}=R$

· Absolute Maximum shear stress

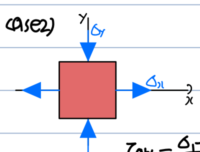
case1)



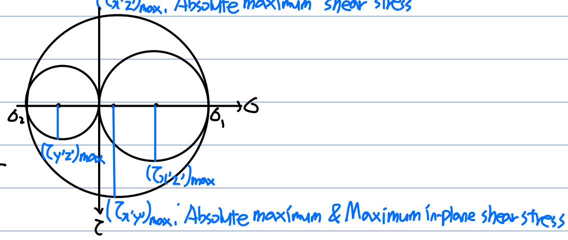
$\tau_{\substack{abs\\max}}=\frac{\sigma_1}{2}$



case2)



$\tau_{\substack{abs\\max}}=\frac{\sigma_1-\sigma_2}{2}$



#9.11 $\sigma_x=150MPa$, $\sigma_y=100MPa$, $\tau_{xy}=75MPa$, $\theta=60°$(clockwise)

$\sigma_{avg}=125MPa$, $R=79.057MPa$

$2\theta_{p_1}=\tan^{-1}\left(\frac{75}{25}\right)=1.249rad$, $\theta=1.047rad$

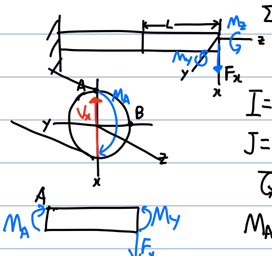
$\theta'=2\theta_{p_1}+2\theta=3.343rad$

$\sigma_{x'}=\sigma_{avg}+R\cos\theta'=47.55MPa$, $\sigma_{y'}=\sigma_{avg}-R\cos\theta'=202.45MPa$

$\tau_{x'y'}=R\sin\theta'=-15.81MPa$

#9.38

$F_x=800N$, $M_y=300N\cdot m$, $M_z=45N\cdot m$, $L=0.45m$, $r=0.025m$



$\Sigma F_x=V_x+800N=0$, $V_x=-800N$

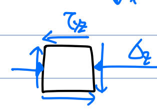
$\Sigma M_y=-L\times F_x+M_y=-60N\cdot m$

$I=\frac{\pi}{4}(0.025m)^4=3.068\times10^{-7}m^4$

$J=\frac{\pi}{2}(0.025m)^4=6.136\times10^{-7}m^4$

$\tau_{yz}=\frac{M_z\cdot r}{J}=1.83MPa$

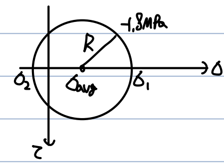
$M_A+0.45m\times800N-300N\cdot m=0$, $M_A=-60N\cdot m$



$\sigma_z=\frac{M_A\cdot r}{I}=4.89MPa$

$\sigma_{avg}=2.445MPa$, $R=3.05MPa$

$\sigma_1=\sigma_{avg}+R=5.445MPa$, $\sigma_2=\sigma_{avg}-R=-0.605MPa$



Ch10. Strain transformation

· Plain-Strain transformation

$\epsilon_{x'}=\frac{\epsilon_x+\epsilon_y}{2}+\frac{\epsilon_x-\epsilon_y}{2}\cos 2\theta+\frac{\gamma_{xy}}{2}\sin 2\theta$

$\epsilon_{y'}=\frac{\epsilon_x+\epsilon_y}{2}-\frac{\epsilon_x-\epsilon_y}{2}\cos 2\theta-\frac{\gamma_{xy}}{2}\sin 2\theta$

$\frac{\gamma_{x'y'}}{2}=-\left(\frac{\epsilon_x-\epsilon_y}{2}\right)\sin 2\theta+\frac{\gamma_{xy}}{2}\cos 2\theta$

· Principal strain (maximum & minimum normal strain)

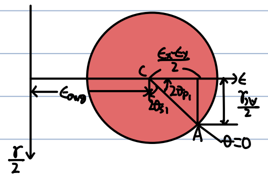
$\tan 2\theta_p=\frac{\gamma_{xy}}{\epsilon_x-\epsilon_y}$, $\epsilon_{1,2}=\frac{\epsilon_x+\epsilon_y}{2}\pm\sqrt{\left(\frac{\epsilon_x-\epsilon_y}{2}\right)^2+\left(\frac{\gamma_{xy}}{2}\right)^2}$

· Maximum in-plane shear strain

$\tan 2\theta_s=-\frac{\epsilon_x-\epsilon_y}{\gamma_{xy}}$

$\frac{\gamma^{max}_{in\text{-}plane}}{2}=\sqrt{\left(\frac{\epsilon_x-\epsilon_y}{2}\right)^2+\left(\frac{\gamma_{xy}}{2}\right)^2}$

$\epsilon_{avg}=\frac{\epsilon_x+\epsilon_y}{2}$



· Absolute maximum shear strain

case1) ϵ_1, ϵ_2 same sign

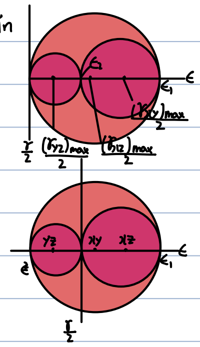
$(1+\epsilon_1)dx \,/\, (1+\epsilon_2)dy$

$\gamma^{abs}_{max}=(\gamma_{x'z'})_{max}=\epsilon_1$

case2) ϵ_1, ϵ_2 opposite sign

$(1+\epsilon_1)dy \,/\, (1-\epsilon_2)dy$

$\gamma^{abs}_{max}=(\gamma_{x'y'})^{in\text{-}plane}_{max}=\epsilon_1-\epsilon_2$



· Strain Rosettes

$\epsilon_a=\epsilon_x\cos^2\theta_a+\epsilon_y\sin^2\theta_a+\gamma_{xy}\sin\theta_a\cos\theta_a$

$\epsilon_b=\epsilon_x\cos^2\theta_b+\epsilon_y\sin^2\theta_b+\gamma_{xy}\sin\theta_b\cos\theta_b$

$\epsilon_c=\epsilon_x\cos^2\theta_c+\epsilon_y\sin^2\theta_c+\gamma_{xy}\sin\theta_c\cos\theta_c$

· Normal strains in 3D

$\epsilon_x=\frac{1}{E}[\sigma_x-\nu(\sigma_y+\sigma_z)]$

$\epsilon_y=\frac{1}{E}[\sigma_y-\nu(\sigma_x+\sigma_z)]$

$\epsilon_z=\frac{1}{E}[\sigma_z-\nu(\sigma_x+\sigma_y)]$

· Material property relationships

Poisson's effect: $\epsilon_{lat}=-\nu\epsilon_{long}$

$G=\frac{E}{2(1+\nu)}$, $K=\frac{E}{3(1-2\nu)}$

· Shear strains in 3D

$\gamma_{xy}=\frac{\tau_{xy}}{G}$, $\gamma_{yz}=\frac{\tau_{yz}}{G}$, $\gamma_{xz}=\frac{\tau_{xz}}{G}$

#10.50 $r=0.015\,m$, $\epsilon_{x'}=-80(10^{-6})$, $\epsilon_{y'}=80(10^{-6})$, $E_{st}=200\,GPa$, $\nu_{st}=0.3$

only shear → $\epsilon_x=0, \epsilon_y=0$ / $\theta=45°$

$\epsilon_{x'}=\epsilon_x\cos^2\theta+\epsilon_y\sin^2\theta+\gamma_{xy}\sin\theta\cos\theta$, $-80(10^{-6})=\frac{1}{2}\gamma_{xy}$, $\gamma_{xy}=-160(10^{-6})$

$G=\frac{E_{st}}{2(1+\nu_{st})}=\frac{200\times10^9}{2(1+0.3)}=7.692\times10^{10}$

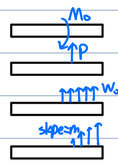
$\tau=G\gamma_{xy}=12.31\,MPa$

$J=\frac{\pi}{2}r^4=\frac{\pi}{2}\times(0.015m)^4=7.952\times10^{-8}m^4$

$\tau=\frac{Tc}{J}$, $T=\frac{\tau J}{c}=\frac{12.31\times10^6 Pa\times7.952\times10^{-8}m^4}{0.015m}=\underline{65.26\,N\cdot m}$

Ch12. Deflection of Beams

Discontinuity Functions



moment: $w=M_0\langle x-a\rangle^{-2}$

Force: $w=P\langle x-a\rangle^{-1}$

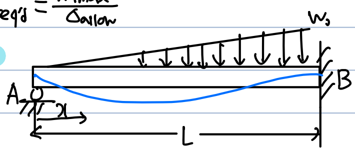
weight function: $w=w_0\langle x-a\rangle^0$

slope weight function: $w=m\langle x-a\rangle^1$

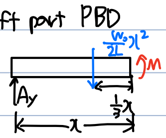
$w\to V\to M=EI\frac{d^2v}{dx^2}$

· $S_{req'd}=\frac{M_{max}}{\sigma_{allow}}$

ex)



Left part FBD

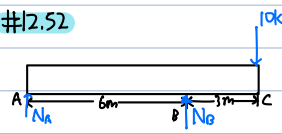


$\Sigma M=M-A_y\times x+\frac{w_0}{2L}x^2\times\frac{1}{3}x=0 \to M=A_y x-\frac{w_0}{6L}x^3=\frac{d^2v}{dx^2}EI$

$\frac{dv}{dx}=\frac{1}{EI}\left(\frac{A_y}{2}x^2-\frac{w_0}{24L}x^4\right)+C_1$, $v(x)=\frac{1}{EI}\left(\frac{A_y}{6}x^3-\frac{w_0}{120L}x^5\right)+C_1x+C_2$

B.C: $v(0)=0$, $v(L)=0$, $\frac{dv}{dx}\Big|_{x=L}=0$

#12.52



$N_A+N_B=10kN$, $6N_B-90kN\cdot m=0$

$N_B=15kN$, $N_A=-5kN$

$w=-5\langle x-0\rangle^{-1}+15\langle x-6\rangle^{-1}-10\langle x-9\rangle^{-1}$

$V=-5\langle x-0\rangle^0+15\langle x-6\rangle^0-10\langle x-9\rangle^0$

$M=-5\langle x-0\rangle^1+15\langle x-6\rangle^1-\cancel{10\langle x-9\rangle^1}$

$M=EI\frac{d^2v}{dx^2}=-5x+15\langle x-6\rangle$

$EI\frac{dv}{dx}=-\frac{5}{2}x^2+\frac{15}{2}\langle x-6\rangle^2+C_1$

$EI\,v=-\frac{5}{6}x^3+\frac{5}{2}\langle x-6\rangle^3+C_1x+C_2$

Boundary condition: $v(0)=0$, $v(6)=0$

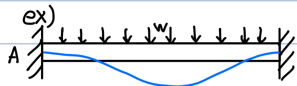
$C_2=0$, $C_1=30\to\theta(x)=\frac{dv}{dx}=\frac{1}{EI}\left[-\frac{5}{2}x^2+\frac{15}{2}\langle x-6\rangle^2+30\right]$, $v(x)=\frac{1}{EI}\left[-\frac{5}{6}x^3+\frac{5}{2}\langle x-6\rangle^3+30x\right]$

$\therefore\theta(9)=\frac{1}{EI}\left(-\frac{5}{2}9^2+\frac{15}{2}\times3^2+30\right)=\underline{-\frac{105}{EI}\,kN\cdot m^2}$

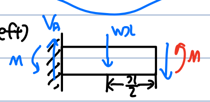
$\therefore v(9)=\frac{1}{EI}\left(-\frac{5}{6}\times9^3+\frac{5}{2}\times3^3+30\times9\right)=\underline{-\frac{270}{EI}\,kN\cdot m^3}$

· Statically indeterminate problem

ex)



FBD(left)



FBD(all)

$\Sigma F_y=V_A+V_B-wL=0$, $V_A=V_B=\frac{wL}{2}$

$\Sigma M=M-x\times V_A+\frac{x}{2}\times wx+M'=0\to M=V_Ax-\frac{w}{2}x^2-M'$

$\frac{dv}{dx}=\frac{1}{EI}\left(\frac{V_A}{2}x^2-\frac{w}{6}x^3-M'x\right)+C_1$, $v(x)=\frac{1}{EI}\left(\frac{V_A}{6}x^3-\frac{w}{24}x^4-\frac{M'}{2}x^2\right)+C_1x+C_2$

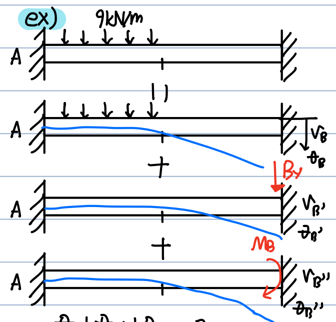
B.C: $v(0)=0$, $v(L)=0$, $\frac{dv}{dx}\Big|_{x=0}=0$, $\frac{dv}{dx}\Big|_{x=L}=0$

$C_2=0$, $C_1=0$, $\frac{V_A}{2}L^2-\frac{w}{6}L^3-M'L=0$, $\frac{V_A}{6}L^3-\frac{w}{24}L^4-\frac{M'}{2}L^2=0$

$\frac{w}{4}L^3-\frac{w}{6}L^3-M'L=0$, $M'=\frac{w}{12}L^2$

$\therefore v(x)=\frac{1}{EI}\left(\frac{wL}{12}x^3-\frac{w}{24}x^4-\frac{wL^2}{24}x^2\right)$

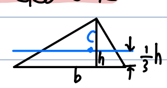
ex)



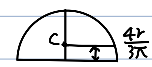
$\theta_B+\theta_B'+\theta_B''=0$

$V_B+V_B'+V_B''=0$

※ Geometric



$I_x=\frac{1}{36}bh^3$



$I_x=I_y=\frac{\pi}{8}r^4$